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Agent based modeling and aggregation of wealth in financial markets

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Tiivistelmä

Tavoitteet

Tässä työssä tutkitaan agenttipohjaista mallintamista osakemarkkinoilla. Erityisesti tutkimuksen kohteena on varallisuuden jakautuminen systeemissä, jossa on suuri määrä keskenään vuorovaikuttavia sijoittajia. Tutkimuksen pääkohteena on kysymys voiko empiirisesti havaittu varallisuuden Pareto-jakauma syntyä ainoastaan markkinamekanismien toiminnan seurauksena. Päätulos tässä työssä on, että se on todellakin mahdollista.

Toteutustapa

Agenttipohjaista mallintamista osakemarkkinoilla on viime aikoina tutkittu runsaasti. Monia tällaisia malleja on ehdotettu lehdissä viimeisen muutaman vuoden aikana. Nämä mallit yrittävät kuvata niitä kollektiivisia ilmiöitä, joita vuorovaikuttavien sijoittajien systeemi tuottaa. Ehdotetut mallit onnistuvat hyvin toistamaan jotkut osakemarkkinoilla havaitut ilmiöt. Kuitenkin monien ilmiöiden alkuperä on vielä epäselvä.

Tässä työssä esitellään muutamia viimeisimpiä ehdotettuja malleja. Niistä valitaan kaksi tarkempaan tutkimukseen ja näitä käytetään varallisuusjakauman tutkimiseen. Työssä osoitetaan, että pienin muutoksin toinen malleista tuottaa Pareto-jakauman, kun taas toisen mallin rakenteessa on sellaisia ongelmia, että sitä ei voida käyttää tähän tarkoitukseen.

Avainsanat

Econophysics, agenttipohjainen mallintaminen, osakemarkkinat, varallisuuden jakautuminen

Abstract

Goals

In this thesis we study agent based modeling in understanding the behavior of financial markets. Particularly we focus on the distribution of wealth in a system of a large number of interacting traders. The question we want to pose is if the empirically observed Pareto distribution of wealth can be created by the trading mechanisms alone. Our main result is that indeed it is possible.

Methods

Applications of agent based modeling in financial markets have been under intense investigation recently. Many such models have been proposed in Statistical Physics literature during the past few years. These models try to capture the collective phenomena that the system of interacting traders produce. They manage to reproduce well some of the observed properties of the financial markets. However, many things still remain unexplained.

In this thesis we review some most recent models. We then take two of the models under closer investigation and apply them to the problem of wealth distribution. It is shown that with a minor modification the other produces the Pareto distribution, whereas the other suffers from some inherent problems that render the model unsuitable for this study.

Keywords

Econophysics, agent based modeling, financial markets, distribution of wealth

Contents

Chapter 1

Introduction

Many phenomena observed in economic complex systems such as the financial markets have striking similarities to the phenomena studied in Statistical Physics. In fact, many powerful tools used in Statistical Physics can be applied for studying systems in Economics and Finance as well. The interaction of Economics and Physics has never been strong, despite the similarities that can be seen in many of the problems under study. However, during the recent years many researchers in the statistical physics community have attacked problems in the field of Economics using the tools of Statistical Physics and non-linear dynamics. In this work we explore this recently emerged field of science called Econophysics.

By applying the tools of Econophysics we show that the recently proposed models in this field show promising potential in explaining some of the peculiarities of the behavior of financial markets. However, we also observe that since the present models are only first attempts in this direction they are far from being perfect. More work needs to be done in identifying the essential mechanism at work in financial markets and correctly including them in the models. We apply and modify two recent models for explaining the distribution of wealth among investors in financial markets. Based on the available models we show that the observed Pareto distribution of wealth can be created solely by the trading mechanism.

One of the first attempts in the direction of trying to find analogies between Physics and Economics was made already in 1936 when Majorana (1942) wrote a pioneering paper in this field. However, for a long time such research remained extremely rare and did not receive much attention. The interest remained low until 1990's when a number of physics researchers inspired by the novel theories of Statistical Physics started to take interest in the complex systems of Economics. Even the term Econophysics was introduced as recently as in 1995 (Stanley et al., 1996).

One such novel theory that could readily be applied to economic systems was the concept of "self-organized criticality" introduced by Bak et al. (1987, 1988). The idea, though quite general, is best understood in light of a specific example. The example chosen by Bak et al. was that of a sand pile. Imagine we start with a clean surface and we deposit one grain of sand at a time. In time the pile grows and its sides become steeper. Eventually, the slope of the sides reaches a critical steepness at which just one more grain would trigger an avalanche.

Bak and coworkers realized that in a complex system it is impossible to predict if a particular grain would trigger an avalanche and which size that avalanche would have. However, the distribution of the avalanche sizes follows a power law, such that all sizes of avalanches occur and the size is only limited by the system size.

Another important property of a system exhibiting self organized criticality is that the critical state is an attractor for the dynamics of the system. No matter what the initial state of the system is, it is always attracted towards the critical state. If we start from a flat pile, as we did above, the pile eventually grows in to the critical steepness. If we start from a state where the steepness of the sides initially exceeds the critical slope, sand will slide off until the critical state is reached.

Since their ground-breaking research the idea of self-organized criticality has been applied to many other natural systems including the size of earthquakes, the spreading of forest fires, X-rays emitted by solar flares, the fluctuations of stock-market prices and many others.

In the complex system of financial markets the price process is produced by the trading activity of a large number of traders. The resulting price fluctuations and occasional crashes are distributed in a similar way to the size of the sand avalanches in the above example. This analogy has been explored among others by Bak himself (1997).

Inspired by the success of the theory of self organized criticality also other new lines of research emerged in the 1990's. For example, many thermodynamical systems display power law behavior near their critical points. This has motivated many researchers to seek analogies between economic systems and standard systems of Statistical Physics. In this thesis, we concentrate on the analogy between financial markets and the dynamics of strongly interacting many body systems.

This thesis is organized in the following way: First in Chapter 2, we introduce some interesting properties of the financial markets that have not been explained by the traditional methods. We also briefly discuss the distribution of wealth in general. Then in Chapter 3, we continue with the introduction of the econophysics approach to complicated systems such as the financial markets. In this Chapter we also introduce the old workhorse of Statistical Physics, the Ising model, which has served as a starting point for many of the econophysics models. In Chapter 4, we put a little more flesh and blood around the general framework and describe some recently used econophysics models in detail. Finally, in Chapter 5, we present the results of this work, namely the data from the models we chose for this study. We also present our extensions of those models and their application to the main goal of the work, that is the study of the development of the distribution of wealth among the investors. This thesis ends with a short summary and discussion in Chapter 6.

Chapter 2

Background

2.1 Properties of financial markets

Natural targets for the application of Physics based ideas in Economics are the financial markets. The stock market, for example, is a system consisting of a very large number of interacting agents, and a large amount of numerical data for the behavior of the prices exists.

For example, we can look at the development of a major stock market index like the Standard & Poors 500 (S&P500), which we present in Fig. 2.1 (a) for a 35-year period ranging from the early 1960's to the late 1990's.

A superficial look at Fig. 2.1 immediately suggests that this must be the result of a random walk process. Indeed, in Finance the idea of a biased Gaussian random walk was presented by Bachelier (1900) already over a hundred years ago. This was actually before Brownian motion became a hot topic in Statistical Physics and Albert Einstein (1905) published his pioneering work on that field. Even today many of the theories for valuing financial contracts are based on the idea that the logreturns are Gaussian

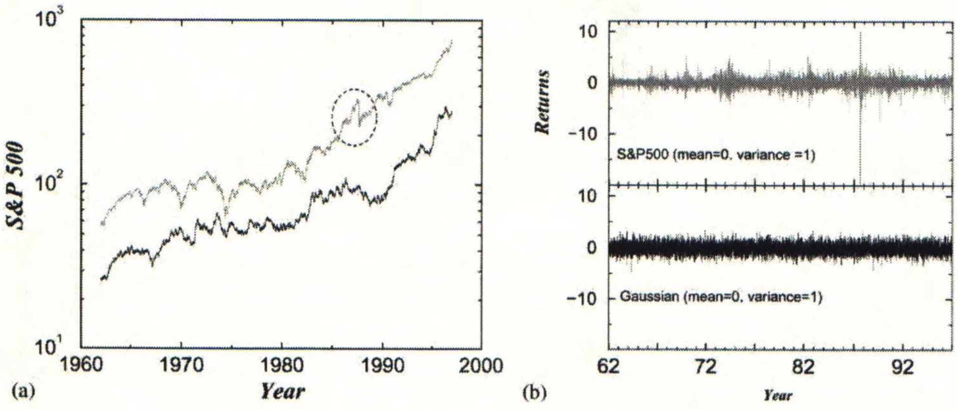


Figure 2.1: The S&P500 index is a normalized sum of the capitalizations of 500 companies traded in New York Stock Exchange. The sharp drop seen in 1987 is the stock market crash of October 19, known as the Black Monday. Figure adopted from Stanley et al. (2001).

random variables. For example the standard form of the celebrated Black-Scholes formula belongs to this class (Hull, 2000).

A closer look, however, reveals that the picture of a Gaussian random walk is not quite accurate. This fact was first pointed out by Mandelbrot (1963) and Fama (1965). In Fig. 2.1 (b) we present returns of the S&P500 over ten minute time intervals and compare them to the corresponding Gaussian random variables. The difference is evident even for the naked eye. The amount of extreme events in the real data is substantially larger than in the Gaussian comparison data. In fact, the largest event is as large as 34 standard deviations, which is virtually impossible in the case of a Gaussian random walk.

It has been established that significant fluctuations in prices are not necessarily related to the arrival of information (Cutler et al., 1989) or to variations in fundamental economic variables (Shiller, 1989). This suggests the high variability in market returns may arise from collective phenomena such as crowd effects or "herding" behavior. One of the main goals of Econophysics is to explain that these catastrophic rare events arise naturally in many interacting many body systems. They are not just inexplicable disasters beyond any possible understanding.

The statistical properties of the time series of stock prices can be quantified for example by computing the autocorrelation functions of returns and volatility and by determining the distributions of these two quantities (Mantegna and Stanley, 2000). These properties have been studied by many authors for both the S&P500 (Mantegna and Stanley, 1995, 1996; Liu et al., 1999; Gopikrishnan et al., 1999, 2000; Töyli, 2002; Michael and Johnson, 2003) and for individual stocks (Plerou et al., 1999; Lillo and Mantegna, 2000; Gopikrishnan et al., 2000; Miccichè et al., 2002) as well as for foreign exchange markets (Dacorogna et al., 1993; Ghashghaie et al., 1996).

The price change in a time series of stocks or a stock index $S(t)$ is usually defined as the change in the logarithm of $S(t)$

$$G(t) \equiv \ln S(t+\Delta t) - \ln S(t) \approx \frac{S(t+\Delta t) - S(t)}{S(t)}, \tag{2.1}$$

where Δt is the sampling time interval. The latter approximation is valid in the limit of small changes in $S(t)$. The volatility can be defined in different ways and we choose the following definition. We take an average of the absolute value of the return $G(t)$ over some time window $T = n\Delta t$

$$V(t) \equiv \frac{1}{n} \sum_{t'=t}^{t+n-1} |G(t')|, \tag{2.2}$$

where n is an integer. Note that in principle this depends both on the sampling time interval Δt and the time window T. The larger the time window T chosen the more accurate the results are. On the other hand, a large T implies poor time resolution. Fortunately, it turns out that the results are not sensitive to the choice of T, see Fig. 2.5. In the simulation results, presented in Chapter 5, we set T equal to one discrete time step. This is done because at least in principle, in simulations one has an arbitrarily large amount of data available and thus the choice is made to obtain maximum temporal resolution. Another frequently used definition for the volatility is the standard deviation of returns.

The autocorrelation function $C(t)$ of a time series such as $S(t)$ is defined as

$$C(t) = E[S(t_0)S(t_0 + t)] - E[S(t_0)]E[S(t)], \tag{2.3}$$

where E denotes an expectation value. When computing the autocorrelation function from given data the expectation value means simply taking an average over all possible t_0. Some authors also call the above defined autocorrelation function autocovariance. It measures the memory the time series has of its own values time t earlier. When there is no memory left $C(t)$ has decayed to zero.

In Fig. 2.2(a) we present the autocorrelation function of returns. It can be seen that the autocorrelation of returns decays exponentially in time. It may be surprising that the returns have any autocorrelation at all. However, the autocorrelation time is so short, only about 4 minutes, that it is difficult to make any money on this. In practice we can say that there are no long range correlations in the returns data.

A more interesting question concerns the autocorrelation of the volatility. Even though the autocorrelation time of returns is very short, volatility is correlated over long periods of time. We present the volatility autocorrelation function in Fig. 2.2(b) for $\Delta t = 1$ min. What is even more interesting is that the decay is not exponential but it goes like a power law. Note that in a log-log scale a power law is a straight line. It should be pointed out here that a power law decay may not have a characteristic scale, or an autocorrelation time in this case, like the exponential decay.

The classical financial theories are based on the assumption that the log

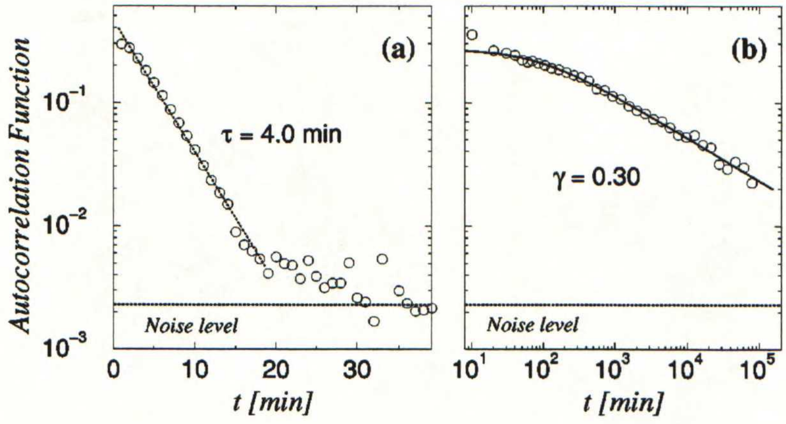


Figure 2.2: (a) The autocorrelation of returns of S&P500 displays an exponential decay $C(t) \sim \exp(-t/\tau)$. (b) The autocorrelation of volatility of the same data decays as a power law $C(t) \sim t^{-\gamma}$. Note the semi-log scale in (a) and the log-log scale in (b). Adopted from Liu et al. (1999).

returns are not just Gaussian random variables, but also identically and independently distributed (i.i.d.). In addition to the observed non Gaussian behavior also the assumption of independence seems to be an oversimplification. Clearly there are some temporal correlations in the observed data.

As was seen above in Fig. 2.1 the distribution of price changes is clearly not Gaussian. Since the price process is influenced by a large number of interacting investors it is reasonable to expect instead a power law like behavior, such that $P(G) \sim G^{-(\alpha+1)}$, where P is the distribution function. The notation $\alpha + 1$ is due to the popular practice of denoting the exponent in the *cumulative* distribution function by α. In Fig. 2.3 we show the distribution of returns of the S&P500 index over one minute time intervals. As is seen in the Figure, the tails of the distribution display a power law behavior with the exponent $\alpha \approx 3$. The same behavior with the same exponent is also seen in the distribution of returns of individual companies (Plerou et al., 1999).

The distribution of returns for longer time intervals than one minute shows

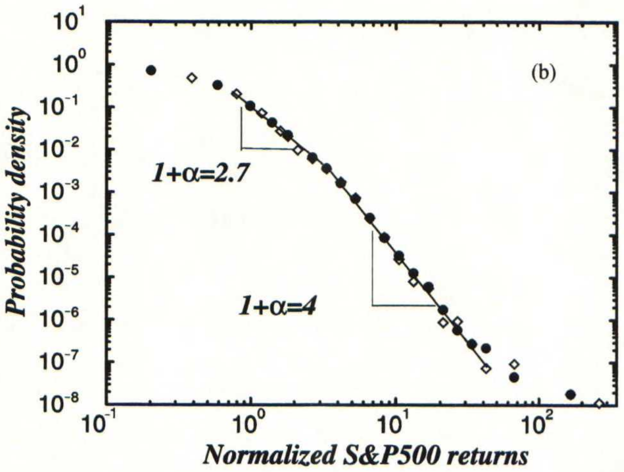


Figure 2.3: The distribution of one minute returns of the S&P500 index. The solid and open symbols denote the positive and negative tails, respectively. Adopted from Stanley et al. (1999).

a slow convergence to a Gaussian behavior. In Fig. 2.4 we show the returns of S&P500 over periods of 4, 8, and 16 days. For time periods roughly longer than two weeks the process seems to be Gaussian. Again, the same behavior has also been observed for individual companies. In this case, however, the non Gaussian behavior seems to persist for time periods up to four years for the positive tail and even longer for the negative tail (Plerou et al., 1999).

We can also look at the distribution of volatility defined above through Eq. (2.2). This we present in Fig. 2.5. A power law like distribution is again seen here in the tail with roughly the same exponent as for the return distribution above. The fact that the dependence of the results on the choice of the time window T is very weak is clearly seen in the Figure 2.5.

Another interesting quantity characterizing the trading process is the volume of trading, measured as the number of shares traded. Also this quantity is seen to display power law like behavior as shown in Fig. 2.6.

It seems that many of the quantities describing the behavior of financial

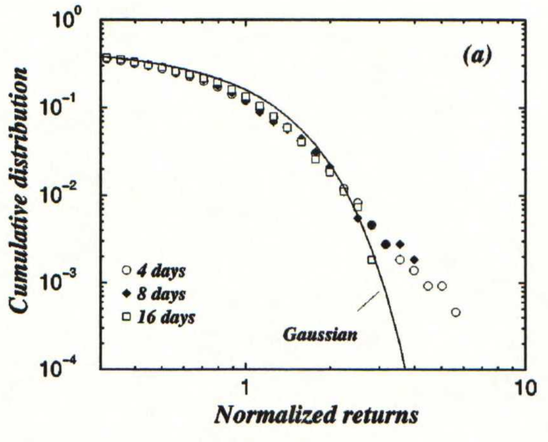


Figure 2.4: The cumulative distribution of returns of the S&P500 index over periods of 4, 8 and 16 days. Adopted from Gopikrishnan et al. (1999).

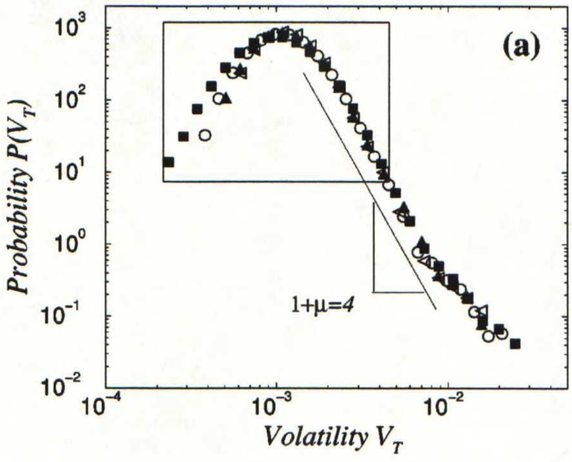
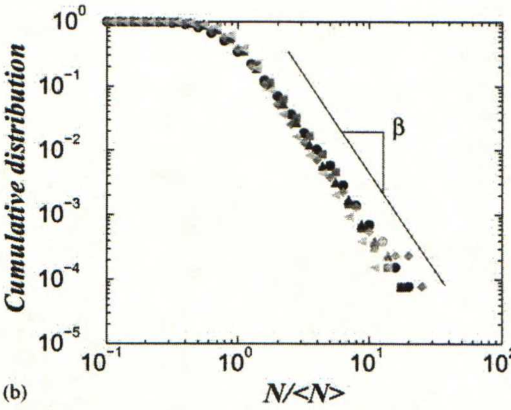


Figure 2.5: The distribution of volatility of the S&P500 index with different time windows T and $\Delta t = 30$ min. For solid squares $T = 120$ min, for open circles $T = 300$ min, for solid triangles $T = 600$ min, and for open triangles $T = 900$ min. Adopted from Liu et al. (1999).



(b)

Figure 2.6: The cumulative distribution of the trading volume for a 30 min period for five different stocks: Exxon, General Electric, Coca Cola, AT&T, and Merck. The power law exponent $\beta \approx 3.4$. Adopted from Gopikrishnan et al. (2000).

markets are well described by power laws. It is worth pointing out here that in general physicist are fascinated by power laws. This is because complex, collective phenomena give rise to power laws which are *universal*. That is they are to a large extend independent of the microscopic details of the phenomenon in question. In the physics literature there are ample examples of this.

This is also related to an important property of power laws, namely scale invariance. The characteristic length scale of a physical system at its critical point is infinite, leading to self-similar, scale-free fluctuations. Similarly a power law distribution is scale invariant in the sense that the relative probability to observe an event of a given size and an event ten times larger is independent of the reference scale. Further discussion about the role of power laws in economics and finance has been given for example by Solomon and Richmond (2002) and Stanley (2003).

The properties presented above and some others are nicely summarized by Cont (2001) as the following 11 stylized facts.

1. **Absence of autocorrelations:** Autocorrelations of asset returns are insignificant, except for very short time scales.

2. **Heavy tails:** The distribution of returns at short time intervals displays power law like tails. In most studies the exponent is seen to be around three. The precise form of the distribution is difficult to determine, however.

3. **Gain/loss asymmetry:** Large drops or crashes are sometimes observed in stock prices and in stock index values, but equally large upward movements do not take place.

4. **Aggregational Gaussianity:** For longer time intervals the distribution of returns becomes more and more like a Gaussian. Thus the shape of the distribution is not the same at different time scales.

5. **Intermittency:** Returns display a high degree of variability at any time scale. This is quantified by the presence of irregular bursts in the time series of a wide variety of volatility estimators.

6. **Volatility clustering:** Measures of volatility display long autocorrelation times. This means that high- and low-volatility periods tend to cluster in time.

7. **Conditional heavy tails:** Even after correcting returns for volatility clustering the residual time series still exhibit heavy tails.

8. **Slow decay of autocorrelation in absolute returns:** The autocorrelation of absolute returns decays slowly as a power law. In different studies the exponent is seen to vary between 0.2 and 0.4. In this work absolute returns are used as a measure of volatility.

9. **Leverage effect:** Most measures of volatility are negatively correlated with the returns.

10. **Volume/volatility correlation:** Trading volume is correlated with all measures of volatility.

11. **Asymmetry in time scales:** coarse-grained measures of volatility predict fine-scale volatility better than the other way around.

2.2 Distribution of wealth

The oldest and also one of the most famous power laws in Economics is probably the Pareto distribution of wealth (Pareto, 1896). The asymptotic tail of the distribution of wealth among individuals in an economy is often described by a power law

$$P(W) \sim W^{-(1+\mu)}, \tag{2.4}$$

where W is the wealth of an individual and again P is a distribution function. The smaller the exponent μ the slower the decay of the distribution and the larger the contrast between the richest and the poorest. More precisely, if the population size is N, the ratio of the largest wealth to the typical (e.g. median) wealth grows as $N^{1/\mu}$.

One can divide the behavior of the distribution into two distinct cases depending on the value of μ. In the case $\mu < 1$ the average wealth diverges. In this case a finite fraction of the wealth is in the hands of a few individuals even when $N \to \infty$. This situation is called "wealth condensation". On the other hand, when $\mu > 1$ every individual only holds a zero fraction of the total wealth (again in the limit $N \to \infty$, of course). Empirically, the exponent μ is seen to be between 1 and 2, (see e.g. (Levy and Solomon, 1997; Dragulescu and Yakovenko, 2001; Solomon and Richmond, 2002) or (Bouchaud, 2001) and references therein).

There have been some attempts to explain this phenomenon using general and simple models. Bouchaud and Mézard (2000) study the distribution of wealth in terms of a very simple model economy, including exchange between individuals and random speculative trading. They find, that their model is able to reproduce the power law wealth distribution. They also see a phase transition as a function of the relative exchange strength between an economy dominated by few individuals and a situation where the wealth is more evenly spread out.

Burda et al. (2002) extend the model of Bouchaud such that they can study open and closed economies. The terms closed and open refer here to a fixed or unlimited total wealth available in the system, respectively. They also observe a power law distribution of wealth. Their system exhibits wealth condensation and in some cases a sizable proportion of the total wealth is seen to be possessed by a single individual. They call this situation the "corruption" phenomenon.

In this work we study the distribution of wealth in some financial market trading models. The purpose of this is to see if the market trading mechanisms alone can produce the Pareto distribution of wealth or if it is a product of some other economic activity.

Chapter 3

Econophysics

3.1 Collective behavior and agent based modeling

The models of Econophysics try to explain the collective emergent properties observed in many systems. Applications have recently been seen in many areas of Economics and human behavior (Bonabeau, 2002). The word emergent property is used for behavior that a number of individuals produce when they interact according to some simple rules. None of the individuals knowingly produce this emergent phenomenon, it simply arises from the collective action of the whole system. A simple every day example of such a phenomenon is a common traffic jam.

The emergent behavior of the system can sometimes be very counter intuitive and the best way to understand it is to use models that start by modeling the behavior of every individual. These so called agent based models can then be simulated on a computer. An example of counter intuitive emergent behavior is that in the above traffic jam example adding one more lane for the traffic in some cases can actually make the traffic jam worse. Using agent based modeling the reason for this can be tracked down. The additional lane might encourage the drivers to switch lanes more frequently and thus slow down the traffic.

Fortunately, in order to produce the correct collective emergent behavior the model does not need to be correct in every detail on the microscopic level. In Statistical Physics the idea of coarse graining is a common tool in modeling complicated interacting systems. Coarse graining means that one simply ignores some microscopic details of the system and only includes the essential mechanisms. Despite this simplification such models can often produce excellent results on the macroscopic or collective level.

In some respects the methods of Econophysics are reminiscent of the ideas of Evolutionary Game Theory (Alexander, 2002). However, there are some important differences. In Econophysics the agents are often interacting with their nearest neighbors, but they are not playing against them. There is also a random component in the dynamics, which can describe not just random information, but also irrational behavior. Computer simulations are a central tool in studying the models of Econophysics, where as in Evolutionary Game Theory analytic mean field approximations are often used. These approximations are, however, unable to capture the important collective phenomena that we are here interested in.

3.2 Econophysics models for financial markets

3.2.1 Overview of models used in literature

Modeling financial markets has been a very popular target for econophysics modeling. Therefore many different models and approaches exist. Here we describe some of the most popular agent based models used for modeling the financial markets.

The Ising model is the most often used prototype model in Statistical Physics and therefore its behavior is extremely well known. Despite its simplicity it displays extremely rich properties. The basic idea in the Ising model based models is that each investor is thought to reside at a vertex of a periodic lattice. They exchange information with their nearest neighbors and also may have a coupling to the collective opinion of all investors in the system. We return to the more detailed description of the Ising model below. Some examples of models that belong to this class are Chowdhury and Stauffer (1999); Bornholdt (2001); Sznajd-Weron and Weron (2001); Iori (2002); Kaizoji et al. (2002); Yamano (2002); Krawiecki et al. (2002).

Another group is the models based on bond percolation models. The idea in bond percolation is that at each time step an investor is chosen at random. Then the investor either takes action and all other investors belonging to the same cluster follow him, or he remains inactive and instead a new bond between him and a randomly chosen neighbor is created joining them to the same cluster. Whenever a cluster takes action all bonds inside it are removed. The name of the model comes from the notion that a cluster that has become so large that it extends throughout the system from one side to the other is called a percolating cluster. Models that belong to this class are for example Eguíluz and Zimmermann (2000) and Focardi et al. (2002), just to name some recent ones.

Also an extremely simple model that has been used as a basis for economic modeling is the minority game (Challet and Zhang, 1997). The idea in this is very simple. At every round each player chooses one of two sides. All of the players who chose the less popular side win. This seemingly simple game can give rise to very rich dynamics and is highly interesting. It manages to capture the essence of a competition situation, since only a minority of the players can win. Some very recent examples of economic models based on the minority game are Giardina et al. (2001); Johnson et al. (2001); Sherrington et al. (2002).

There is one more type of model that we would like to mention here. This is a model in which the investors are divided into two classes: the fundamentalists and the noise traders (Shleifer and Summers, 1990). The fundamentalist are assumed to have knowledge of the fundamental price of the asset and they choose to trade whenever the market price differs from this fundamental price. The noise traders, on the other hand, do not believe in the fundamental price, but rather try to identify trends and also rely on other traders as a source of information. Depending on the model, there might be some kind of mechanism allowing investors to move from one of these groups to the other. Even though this idea has been incorporated into some of the models that belong to the other classes described above the most basic model using this concept is probably the model proposed by Lux and Marchesi (1999).

3.2.2 The Ising model

The econophysics models that have been most frequently applied to financial systems, and which are also the focus of this theses, are based on the Ising model. The Ising model (Ising, 1925) is the most basic model in Statistical Physics. Its popularity is based on its simplicity and on the fact that it is the only model that displays a complicated phase diagram and is still analytically solvable in some special cases.

In the Ising model the system considered consists of an array of N fixed points called lattice sites that form a d-dimensional periodic lattice. The lattice may be, for example, cubic or hexagonal. At each of these lattice sites we have a spin variable s_i, which is a number that is either $+1$ or -1. There are no other degrees of freedom in the system and thus a set of numbers $\{s_i\}$

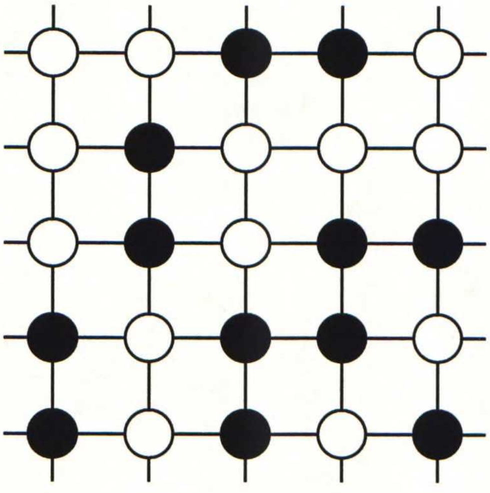


Figure 3.1: The Ising model in a two dimensional square lattice. The open and solid circles denote the two possible states of the spin variable sometimes also called the spin up (+1) and spin down (−1) states.

specifies the configuration of the system. In Fig. 3.1 we present a schematic representation of the system in a two dimensional square lattice.

In the language of Economics we might consider that each spin represents one investor. The value +1 represents a positive investor, who is willing to buy stocks and the value −1 an investor willing to sell.

The spin variables interact with their nearest neighbors such that the energy of the system can be written as:

$$E(\{s_i\}) = -\sum_{\langle i,j \rangle} \epsilon_{ij} s_i s_j - H \sum_{i=1}^{N} s_i. \tag{3.1}$$

Here the interaction energy ϵ_{ij} and the external magnetic field H are given parameters. The symbol $\langle ij \rangle$ denotes nearest neighbors. Thus the first sum contains $\gamma N/2$ terms, where γ is the number of nearest neighbors at each site, also called the coordination number of the lattice. For example in the case of a two dimensional square lattice presented in Fig. 3.1 the coordination number $\gamma = 4$. In the simplest form of the model all ϵ_{ij}'s are equal and denoted only as ϵ. The case $\epsilon > 0$ corresponds to a ferromagnetic system and in this case all the spin variables tend to be equal since in this way the energy of the system is minimized. On the other hand, the case $\epsilon < 0$ corresponds to an antiferromagnetic case in which all spins like to have an opposite sign compared to their nearest neighbors.

Again, in terms of an economic system the energy can be considered as a disagreement function. Every agent likes to agree with his contacts, or neighbors. He might also have a tendency either to agree or disagree with the average opinion of all investors, as represented by the second term in Eq. (3.1).

In a two dimensional lattice, such as the one presented in Fig. 3.1, all the thermodynamic properties of the Ising model can be solved analytically. One such property that is of interest from the point of view of the economics modeling is the ordering taking place in the system. This can be quantified using the concept of magnetization defined as

$$m(H,T,t) = \frac{1}{N}\sum_{i=1}^{N} s_i(t), \tag{3.2}$$

which is a time dependent quantity as in a nonzero temperature all s_i's keep fluctuating. Thus the magnetization is just the average value of all spins, or all opinions in an economic model, and varies between -1 and 1. This quantity will later be associated with a price process in the economic models.

We know from Statistical Physics that nature tends to minimize the free energy of the system. The free energy is a sum of the contributions of energy (3.1) and an entropy term: $F = E - TS$. The second term represents the part of energy that is unavailable because of the basic laws of thermodynamics. Suffice it to say here that it is connected to the ordering of the system. At low temperatures the second term is insignificant and the free energy is minimized when ever the energy E is minimized and thus a state where all spins point in the same direction emerges. However, the higher the temperature T the more significant is the contribution of the entropy S. Entropy is maximized when there is no order in the system and thus at high enough temperatures the entropy term wins and the system becomes disordered, i.e. on average $m = 0$. The states where the time average of m is zero or non-zero are also called different phases of the system.

In the case of the square lattice presented in Fig. 3.1 and in the limit of an infinitely large system and when the external magnetic field in Eq. (3.1) goes to zero the phase transition from the ordered state to the disordered state takes place at a critical temperature $k_B T_c = 2\epsilon/(\sinh^{-1}(1)) \approx 2.27\epsilon$, where k_B is the Boltzmann constant (Kramers and Wannier, 1941a,b). In a higher dimensional cubic lattices the corresponding values are 4.51ϵ and 6.68ϵ in three and four dimensions, respectively.

At temperatures below T_c the system orders. Both possibilities, $m > 0$ and $m < 0$, are equally likely and nothing distinguishes between them. The system chooses one of these states by chance and therefore we say that a spontaneous symmetry braking takes place. In the thermodynamic limit, that is in an infinite system, this also leads to ergodicity breaking, meaning that depending on the initial state the system ends up in either $m > 0$ or $m < 0$ state and cannot switch to the other state. It is worth noting, that the Ising model based economic models utilizing heat bath dynamics have the (artificial) temperature set at below T_c. In the heat bath dynamics employed in most cases the dynamics of the system simulates the behavior a physical system would have at a certain temperature. The details how this is done in practice are given in Section 4.1. However, because the systems always have a finite size, ergodicity breaking does not take place and the system spontaneously fluctuates between the $m > 0$ and $m < 0$ states.

It is well known that the properties of the Ising model depend on the dimension of the system (Goldenfeld, 1992). As already inferred above, in two dimensions all the properties of the system can be analytically solved (Onsager, 1944). In three and four dimensions the rich and complicated properties of this system can only be explored through numerical methods or computer simulations. In higher dimensions mean field approximations can be used. Most of the Ising model based models in Econophysics are based on the two dimensional version of the model and only a few attempts (Yamano, 2002) have been made to investigate the effect of dimensionality in them. Yamano's results suggest that the results are quantitatively different in higher dimensions, but the important qualitative features of the model remain unchanged.

Chapter 4

Models

4.1 Direct application of the Ising model to Finance

The most simple application of the Ising model to financial markets is probably the model introduced by Bornholdt (2001). It attempts to capture two major forces that influence an investor when he is making his investment decision: (1) "Do what your neighbors do", which leads into herding behavior among the investors and (2) "Do what the minority does", as often followed by traders with knowledge about the fundamental value of stocks. The global coupling, point two above, effectively destabilizes local agent views depending on the size of the majority. The resulting frustration between seeking agreement with neighbors locally, but escaping the majority globally, causes dynamics, which is at times fairly calm with some intermittency and at times displays phases of chaotic behavior. In particular, this occurs at temperatures below the critical temperature of the Ising model. The strengths of this model lie in its simplicity rather than in the accuracy with which it reproduces the empirically observed features of financial markets.

In this model the spins s_i represent the views of each investor: $+1$ means that the investor is willing to buy and -1 that he is willing to sell. The Hamiltonian of the system is written as

$$E_B(\{s_i\}) = -\sum_{\langle i,j \rangle} \epsilon_{ij} s_i s_j + m\alpha \sum_{i=1}^{N} C_i(t) s_i, \tag{4.1}$$

where $\alpha > 0$ is a parameter and $C_i(t)$ is a time dependent strategy of agent i, and can take on one of two values $+1$ or -1. Besides the introduction of these two additional factors the only difference between this and the Ising Hamiltonian (3.1) is that the external magnetic field H has been replaced with the magnetization of the system m, given by Eq. (3.2) above. As stated above, in the language of Economics the energy given by Eq. (4.1) can be thought of as a disagreement function. It measures the agents disagreement with his neighbors and the world, and every agent would like to minimize his disagreement function. The strategy choice $C_i = 1$ represents a fundamentalist strategy, the agents tendency to be in the minority, point (2) above. The choice $C_i = -1$ on the other hand represents a noise trader, who wants to go with the crowd. Bornholdt allows a transition between these two strategies by the rule

$$C_i(t+1) = -C_i(t), \quad \text{if } \alpha s_i(t) C_i(t) \sum_{j=1}^{N} s_j(t) < 0. \tag{4.2}$$

This means that a change in strategy does not come free but at a cost. This cost, Bornhodt argues, reflects for example the effort to obtain information, the potential risk as it affects prospective future returns, etc.

The dynamics of the model follows a simple heath bath dynamics using the Kawasaki Monte Carlo algorithm (Kawasaki, 1966a,b,c). One site, or an agent, is picked at random and the spin flipped with a probability p

$$p = \frac{1}{2}[1 - \tanh(\frac{1}{2}\beta\delta E_B)], \tag{4.3}$$

where δE_B is the change in the energy, or total disagreement given by Eq. (4.1) resulting from a spin flip, and $\beta = 1/k_B T$ is the temperature, which of course is a somewhat artificial parameter in an economic model. Thus the states of the system are distributed according to the Boltzmann distribution. One time step is taken to be equal to one Monte Carlo step, that is equally many random attempts as there are agents in the model. Thus in one time step, in average, one attempt is made to flip each spin.

Bornholdt interprets the magnetization, or the average opinion of the investors, m as a measure of price and studies the change in its logarithm as the return. The fluctuations of the thus obtained price exhibit a power law distribution, but the exponent is not in quantitative agreement with empirical observations. The model also reproduces the volatility clustering as measured by the volatility autocorrelation function, but Bornholdt makes no attempt to compare this quantitatively with empirical data.

This simple model has also been studied by Yamano (2002) in three and four dimensional hypercubic lattices. He finds that the results remain qualitatively the same in those cases.

4.2 Ising model with an explicit price process

A modification of the simple Bornholdt model is introduced by Kaizoji et al. (2002). They point out that simply interpreting the magnetization as the price is not very realistic. Instead, as a modification to the Bornholdt model they propose an explicit price formation process, which is based on the balance of supply and demand in the markets.

Instead of using the strategy variable $C_i(t)$ they divide the traders into a fixed number of fundamentalists and interacting traders. The fundamentalists have knowledge of a fundamental price $p^*(t)$ and they trade on the difference of the market price $p(t)$ and $p^*(t)$. Their buying or selling order is given by

$$x_F(t) = a\, n_F\, (\ln p^*(t) - \ln p(t)), \tag{4.4}$$

where n_F is the number of fundamentalist traders and a is a parameter characterizing the strength of their reaction on the discrepancy between the fundamental price and the market price.

The interacting traders behave as in the standard Bornholdt model and their supply or demand is give by

$$x_I(t) = b\, n_I\, m(t), \tag{4.5}$$

where b is a parameter, n_I is the number of interacting traders, and $m(t)$ is again the magnetization, or the average view of the investors, defined by (3.2). Then at every step the market price is simply set such that the supply and demand are in balance

$$x_F(t) + x_I(t) = 0. \tag{4.6}$$

Solving this equation for $\ln p(t)$ also yields the trading volume at each time step.

The dynamics of the model is taken to be the same heath bath dynamics used in the original Bornholdt model presented above.

The exponent of the distribution of logreturns is in better agreement with experimental data than in the original Bornholdt model, though the result is still not quite accurate. Volatility clustering is also observed in this model, but the power law nature of the volatility autocorrelation function is not reproduced.

4.3 The Iori consultation model

The model of Iori (2002) is the only one of the models presented here that does not use heat bath dynamics and therefore has no artificial temperature in it. Instead she proposes dynamics in which the agents consult each other and form their opinions based on the opinions of their neighbors. The influence of the environment Y on investor i is described by

$$Y_i(\tilde{t}) = \sum_{\langle i,j \rangle} J_{ij} s_j(\tilde{t}) + A\nu_i(t), \tag{4.7}$$

where $s_j(\tilde{t})$ is the temporary choice of investor j and it may get updated from a consultation round to the next. The agents are assumed to sit in a two dimensional square lattice and the summation is only over the nearest neighbors as denoted by $\langle\rangle$. The interaction parameters J_{ij} are all taken to be equal and set to J. The idiosyncratic noise $\nu_i(t)$ is a random number uniformly distributed on the interval $[-1, 1]$. It describes any external random information the investor may have. The amplitude of the idiosyncratic noise A is a parameter. Thus Y describes the aggregate information the agent is receiving from his environment.

At each consultation round every agent updates his views according to

$$s_i(\tilde{t}) = \begin{cases} 1, & \text{if } Y_i(\tilde{t}) \geq \xi_i(t); \\ 0, & \text{if } -\xi_i(t) < Y_i(\tilde{t}) < \xi_i(t); \\ -1, & \text{if } Y_i(\tilde{t}) \leq -\xi_i(t), \end{cases} \tag{4.8}$$

where every agent has his individual threshold $\xi_i(t)$ for taking action, and these are initially chosen from a Gaussian distribution with zero mean and unit variance. The view of each agent is updated sequentially. The consultation rounds are continued until each $s_i(\tilde{t})$ converges to its final value $s_i(t)$.

When convergence is achieved the supply Z and demand D are determined in the following way

$$D(t) = \sum_{i:s_i(t)>0} s_i(t), \tag{4.9}$$

$$Z(t) = -\sum_{i:s_i(t)<0} s_i(t). \tag{4.10}$$

The market price P is updated according to

$$P(t+1) = P(t)\left(\frac{D(t)}{Z(t)}\right)^{\alpha}, \tag{4.11}$$

where

$$\alpha = a\frac{Z(t)+D(t)}{N}, \tag{4.12}$$

where N is the total number of traders and a is a parameter controlling the magnitude of the price variations. Subsequently also the trading thresholds are updated according to

$$\xi_i(t+1) = \xi_i(t)\frac{P(t)}{P(t-1)}. \tag{4.13}$$

This is motivated by the assumption that the trading threshold is influenced by transaction costs, which, Iori argues, are proportional to stock prices.

The model reproduces volatility clustering and the behavior of the volatility autocorrelation function is close to a power law. The decay rate of the volatility autocorrelation function seems to be in good agreement with empirical results. Also the power law tails of the return distribution are present and the distribution of returns is in good qualitative agreement with real markets. This distribution is seen to cross over to a Gaussian distribution when returns at longer time intervals are considered.

4.4 The Super-spin Ising model

Chowdhury and Stauffer (1999) have a little different interpretation for the spin variables s_i. In their model s_i does not represent just one investor, but an investment agency. The investment agencies may have different number of clients and to model that Chowdhury et al. let each superspin assume one of three possible values: $-|S_i|$, 0 or $+|S_i|$. The magnitudes of the spins $|S_i|$ are drawn from a predetermined distribution.

The interaction between these superspins is assumed to be global instead of just the nearest neighbor interactions in other models. Thus they write the interaction energy as

$$E(\{s_i\}) = -J \sum_i \sum_{i \neq j} s_i s_j - \sum_i s_i h_i. \tag{4.14}$$

In addition to the interactions among the superspins they also include a local magnetic field h_i, which represents the individual bias of each investment agency. Also in this model the dynamics is taken to follow the heath bath dynamics presented above.

Their model reproduces the power law distribution of returns, but the exponent seems to reflect the choice of the distribution of the superspin magnitudes $|S_i|$. They give no results for other properties of the produced price process such as the volatility autocorrelation function.

4.5 The random interaction model

In their model Krawiecki et al. (2002) emphasize the fact that the interaction network among the investors is not static, but can change in time. Thus their model does not assume any fixed topology for the network of investors. They write the interaction energy of the system as

$$E(\{s_i\}) = -\sum_{i,j} A_{ij}(t) s_i s_j + \sum_{i=1}^{N} h_i(t) s_i, \tag{4.15}$$

where the local field h_i reflects external influences which might differ from agent to agent and again s_i is the buying (+1) or selling (−1) decision of agent i. Both the interaction strengths A_{ij} and the external field h_i change randomly in time,

$$A_{ij}(t) = A\xi(t) + a\eta_{ij}(t), \tag{4.16}$$

$$h_i(t) = h\zeta_i(t), \tag{4.17}$$

where they assume that $\xi(t)$, $\eta_{ij}(t)$ and $\zeta_i(t)$ are i.i.d. random variables uniformly distributed in the interval $(-1, 1)$. Thus no lattice needs to be assumed since $A_{ij}(t)$'s are nonzero for all pairs ij. The simple heath bath dynamics explained in connection with the Bornholdt model in Section 4.1 is used in this model as well.

Their main point is that the fluctuating interactions make the system fluctuate between $m > 0$ and $m < 0$ states even in the thermodynamic limit. Also, the details of the interactions among pairs of agents, and the details of the variation of the mean interaction strength are unimportant for the qualitative results.

The power law nature of the returns is well described by this model even if they set $h = 0$. The exponent of the distribution seems to be strongly dependent on the value of the parameter A. The model also displays volatility clustering, although they do not study this in quantitative detail.

Chapter 5

Results

5.1 The models chosen for this study

We have chosen two of the models presented above for studying the distribution of wealth of the investors in them. The first is the Iori model and the second is the model of Kaizoji. These two models were chosen, because in them the wealth of the agents is readily available or can easily be accounted for.

5.2 The Iori model

5.2.1 The original model

A short description of this model was given above in Section 4.3. Some details of the model are not explained in the original article. First of all, the value of the parameter a in Eq. (4.12) is not given. This parameter sets the scale of the variations of price in the model. If we study normalized price fluctuations the exact value is, of course, unimportant. However, Eq. (4.13) couples the trading thresholds to the absolute price level and thus this parameter is meaningful. We tried different values for this parameter and in the case of the below results we have arbitrarily set it to the value of 2.0.

Secondly, Iori gives no explanation whether she clears the markets before or after the price update. If this is done before, the market maker makes money because he always buys (sells) just after a price decrease (increase). Thus, the market maker ends up absorbing all the money out of the markets. If the price adjustment is done afterwards the opposite thing happens and the market maker is eventually bankrupt. In order to stabilize the system we have done the market clearing at 'half way' through the price update. That is all trades are executed at a price half way in between the new price and the old price.

In the simulations the relevant parameters of the model were chosen to be the same as those used by Iori and are as follows. The size of the lattice of investors was set at 100×100. All agents were initially given 100 units of cash and 100 units of stock. Market maker was endowed with both cash and stock an amount ten times the amount held by all agents together. The initial market price $P(0)$ was set at unity. The initial value of the variance of the trading thresholds was set at $\sigma_\xi^2(0) = 1$. The coefficient A in Eq. (4.7) was fixed at the value 0.2. We ran the simulation for $2 \cdot 10^6$ time steps. This can be done in approximately six hours in a Pentium IV computer. In Fig. 5.1 we present a short sample of the behavior of the price in the simulations.

The model displays volatility clustering. This can be seen in the volatility autocorrelation function presented in Fig. 5.2. Iori claims that this should be a power law. As can be seen in the Figure and also confirmed by data using other parameter sets, our data shows that this is not the case. However, the initial decay is very close to a power law. If one has insufficient statistics and the data quality is thus not good enough, as seems to be the case with

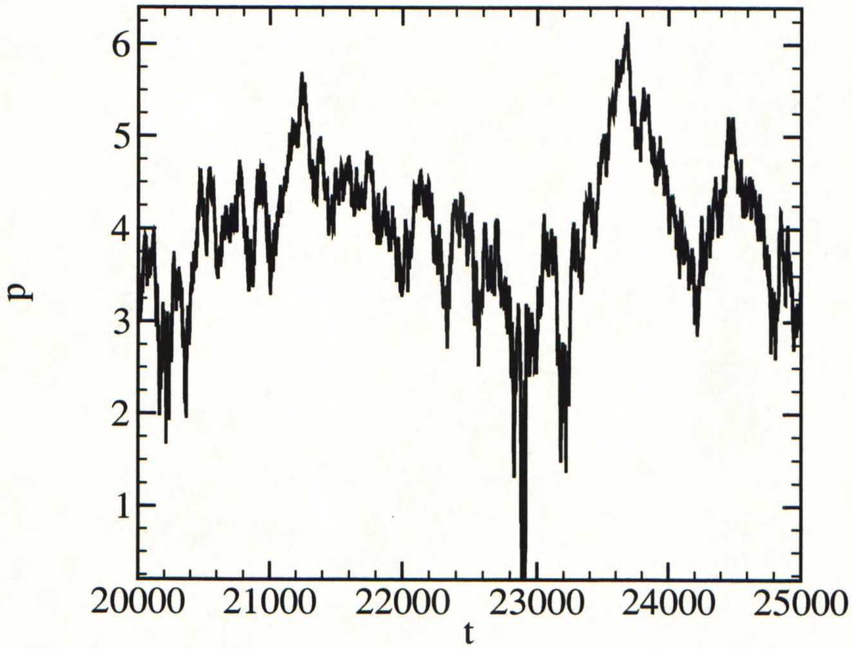


Figure 5.1: Typical behavior of the price in the Iori model.

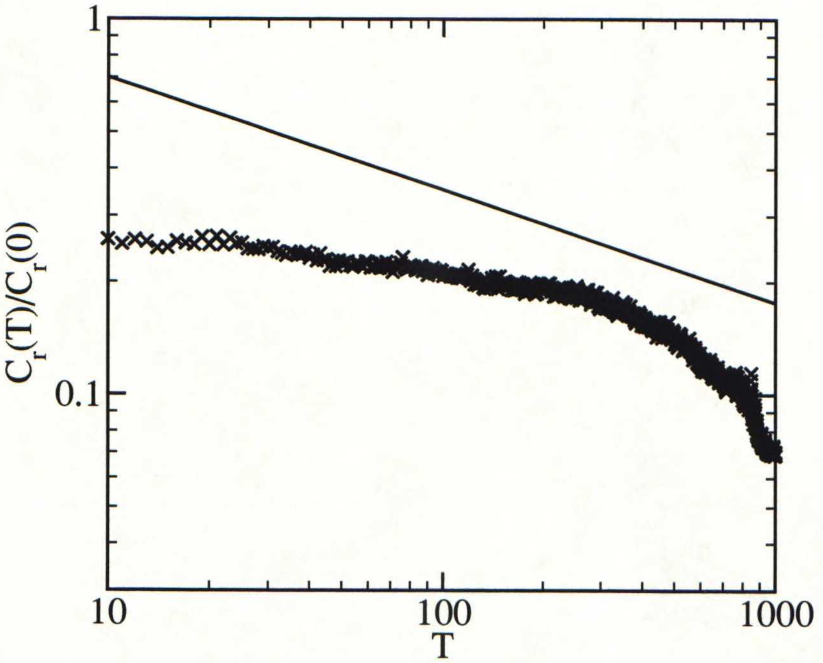


Figure 5.2: The normalized volatility autocorrelation function in the Iori model. The solid line shows the power law $t^{-0.3}$.

the data Iori presents, this can easily be mistaken for a power law. Also, our decay seems to be slower than the $t^{-0.3}$ observed by Iori. The decay rate can be tuned by changing the parameter a, whose value Iori does not give. We have tried different values and qualitatively the results remain the same, no power law is seen. Thus by just tuning this one parameter it is not possible to produce a power law decay in the volatility autocorrelation function.

In the distribution of short time returns a power law behavior is clearly seen. The data in the Fig. 5.3 was taken between $1.5 \cdot 10^6$ and $2.0 \cdot 10^6$ time steps. The exponent is about 5, whereas in Iori's results it was 3, in agreement with empirical results. However, the value of the exponent was seen to be smaller at earlier times and its value can be changed by changing some of the parameter values. Since Iori does not give the exact parameter values she has used nor does she mention at which time she observed the distribution we did not try to tune the model to reproduce the exponent 3. Besides, that would not make any difference in the qualitative behavior of the model. What is important is that a power law is seen here.

We also studied the evolution of the distribution of returns at longer time intervals. In Fig. 5.4 we show the distribution of returns at time intervals 20000 time steps. We see that at such long time intervals the distribution has almost completely converged to a Gaussian distribution. For comparison also the one step return distribution presented in Fig. 5.3 is included in Fig. 5.4. Note the scales at the axis. Since the probability density of the Gaussian distribution goes as $P \propto \exp(-r^2/\sigma^2)$ plotting it at a semi-log scale against r^2 should produce a straight line.

Our main goal in this thesis is to study the distribution of wealth in agent based modeling of financial markets. In Figure 5.5 we present the distribution of wealth in the Iori model after almost two million time steps. Originally at time zero the wealth was evenly distributed among the agents. In the Figure it can be seen that the distribution is perfectly Gaussian. Since empirically

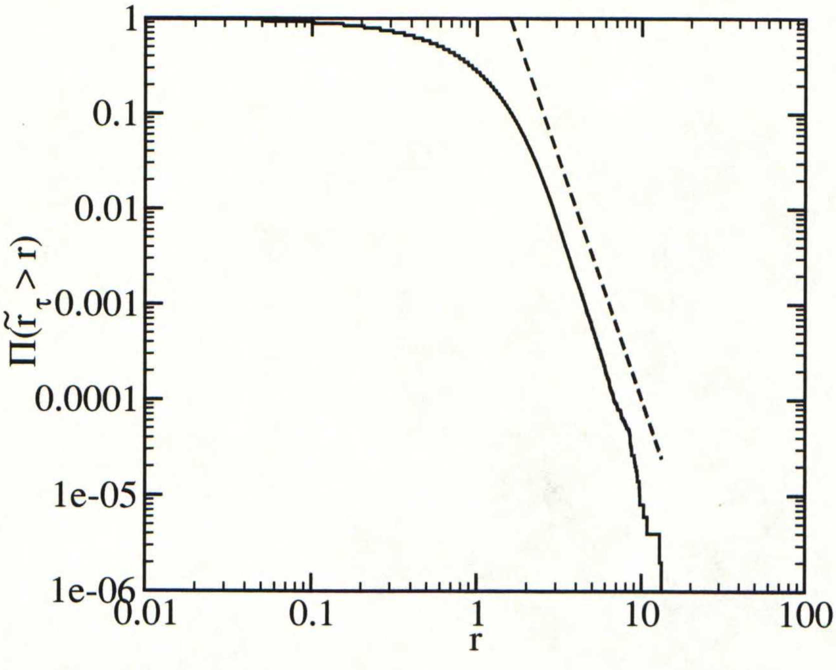


Figure 5.3: The cumulative distribution function of one step returns in the Iori model. The dashed line shows the power law r^{-5}.

it has been observed that the wealth distribution is a power law it seems that the model is lacking some critical ingredient.

In addition to not producing a power law distribution of wealth the model also has some other unrealistic properties. For example because the volatility is controlled by the level of trading thresholds, and the trading thresholds are connected to the price level, periods of high volatility and periods of low price are perfectly correlated. Also the role of the market maker in the model is somewhat dubious. Although the market maker can be argued to be connected to a group of fundamentalist traders as in the Kaizoji model presented above in Section 4.2.

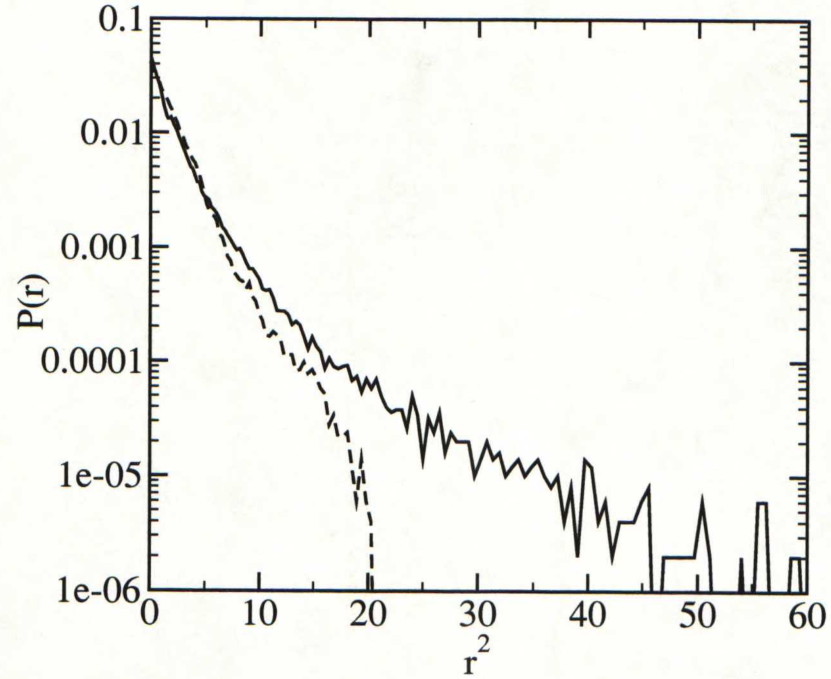


Figure 5.4: The distribution function of 20000 step returns (broken line). For comparison the solid line represents the one step return distribution function.

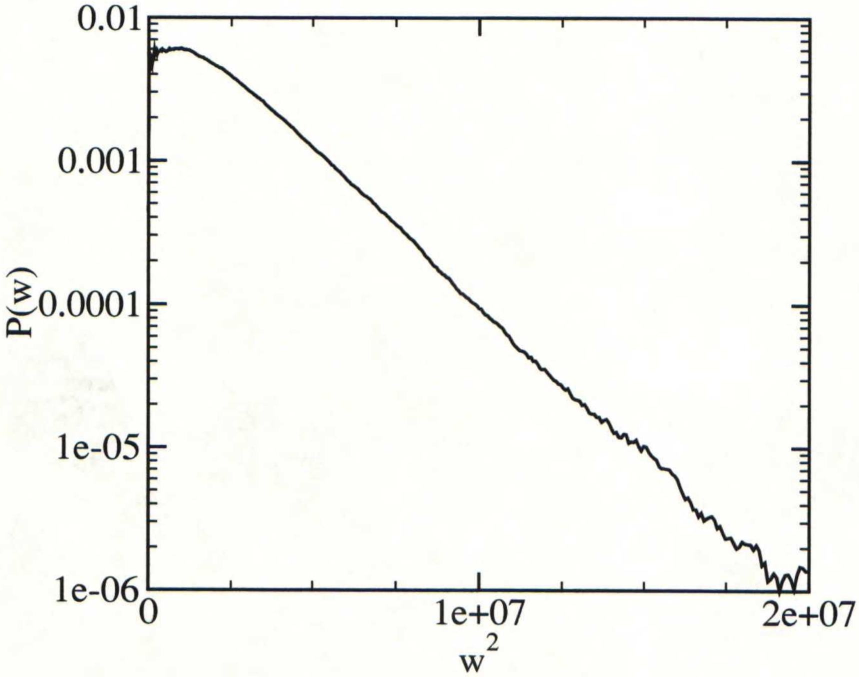


Figure 5.5: The distribution of wealth in the Iori model.

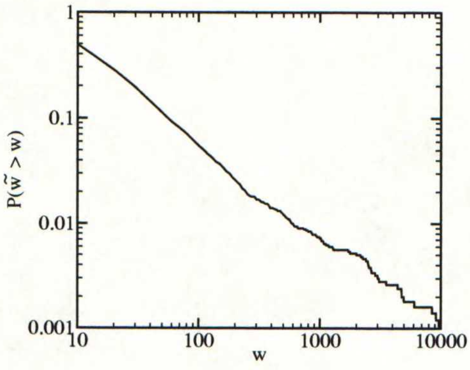
5.2.2 Modified Iori model

Since we are interested in the distribution of wealth we concentrate on improving this property of the model. In doing so we target the trading process itself, because this is what is responsible for the flows of wealth. In the Iori model the agent always buys or sells one stock at a time. This is, of course, not very realistic. Instead, we propose that the agents always buy or sell stocks an amount equivalent in value to fixed proportion of his wealth. This is probably not a quite accurate description of the real trading behavior either. However, it establishes some link between the size of the trades and the individual investment needs and power of each agent. Therefore it certainly seems more plausible than the original idea of trading just one stock at a time, which makes the size of each trade proportional to the absolute market price. This slight alteration does not really make the model any more complicated, but it turns out that this is the critical ingredient that the model is missing to produce a power law distribution of wealth.

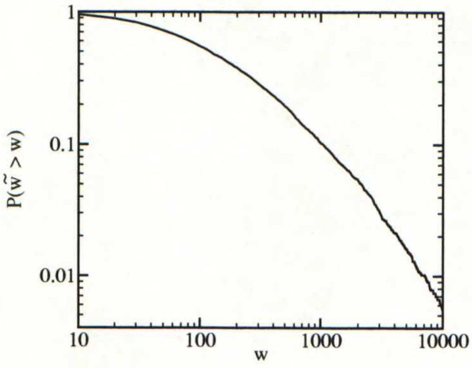
We further motivate this modification with the following example. Consider an investor who invests a proportion p of her wealth. For her investment she gets a return r. She repeats the investment n times and thus her final wealth W is given by

$$W = \prod_{i=1}^{n}[1 + (e^{r_i} - 1)p], \tag{5.1}$$

where r_i is the return on her i:th investment. The return r_i is a random variable and if we use for it the distribution of the Iori model shown in Fig. 5.3 we obtain the distribution shown in Fig. 5.6 (a) for the final wealth W. This is clearly a power law, the irregularities at the tail are just bad statistics. However, the effect of this modification in the real simulation is not this straight forward and trivial. Remember, that in this example we used the one step distribution for r_i, which is not stable. In the long run it converges into the Gaussian distribution. If the stock is held for a longer period of time



(a) One step Iori distribution for r_i.



(b) Gaussian distribution for r_i.

Figure 5.6: The probability density of the simple proportional investing example. Here we have set $p = 0.1$ and $n = 50$

the proper distribution for r_i is the Gaussian distribution. If we draw the r_i from the Gaussian distribution instead we obtain the distribution shown in Fig. 5.6 (b) for the final wealth W, which clearly is *not* a power law. In addition, in the simulations the returns each individual investor receives are not independent. Thus, always investing a fixed proportion of ones wealth does not necessarily produce a power law distribution of wealth. Therefore, it should be fully appreciated that this is not a trivial modification.

Our modified version of the model suffers from the same instability than the original model. There is no guarantee that either supply or demand could not become zero. Making investors always invest or sell a fixed proportion of their wealth only enhances this problem. Especially if $a \gtrsim 2.0$ it becomes increasingly likely that a random fluctuation throws the price very high and then the trading thresholds become so high that sooner or later either supply or demand becomes zero. According to the price update rule this would produce an infinitely large change in the price. Whenever this happens we just redo the round until at least one buyer or seller appears, where there otherwise would have been none. However, since these events are extremely rare ignoring them does not affect the results too much. In fact, when the system is made large enough this stops from happening all together. With the parameters used a system of 40000 agents displays no infinite events.

In our simulations all the parameters of the model were set at the same values, which we used in the original model. The proportion of his wealth the investor buys or sells at one time was set to 0.001. The value chosen does not seem to have any fundamental importance, other than a smaller value results in a slow evolution of the wealth distribution and a large value can result into very large orders, which can destabilize the price process. The data is taken between $5 \cdot 10^5$ and $1 \cdot 10^6$ time steps. In Fig. 5.7 we present the volatility autocorrelation function of our modified model. Qualitatively it is similar to the one observed in the original model. Only the decay rate seems to be somewhat faster than in the original model, but still no power law is seen here.

Fig. 5.8 shows the cumulative distribution of one step returns in the modified model. Again, there is no qualitative change compared to the original model. The decay exponent, 2.2, is in good agreement with empirical findings.

Finally, in Fig. 5.9 we present the distribution of wealth in the system in the end of the simulation run. In here we see a remarkable difference compared to the original model. Instead of the Gaussian distribution the tail of the distribution of wealth is clearly a power law. The decay exponent, 4.1, is somewhat larger than what is seen empirically, but it might not have fully converged in the end of the run. At earlier times the distribution is also seen to obey a power law, but with a larger exponent. Thus running the simulation even further might still lower the exponent. In any case, the exponent is clearly larger than unity and thus wealth condensation does not take place in this system.

We also made attempts to remedy some of the other unrealistic properties

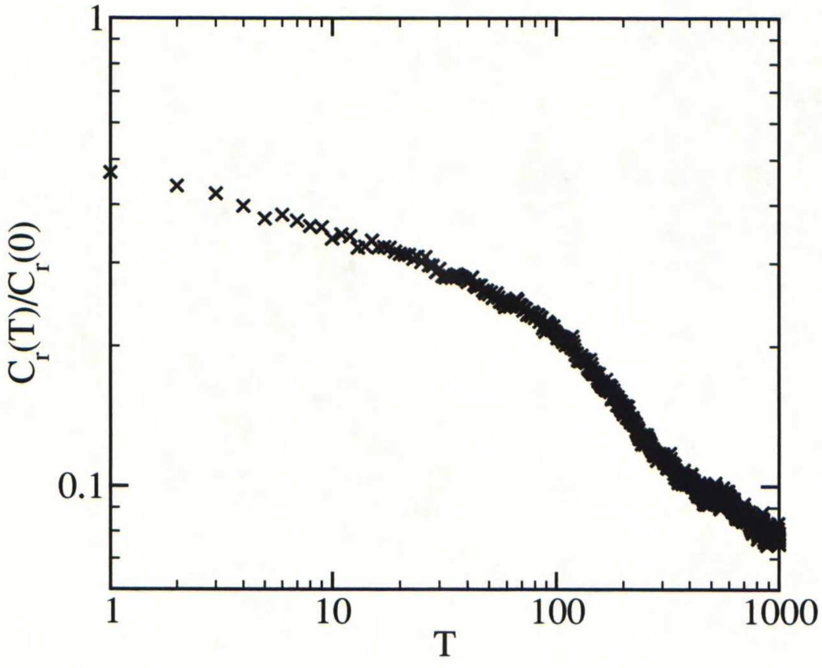


Figure 5.7: The normalized volatility autocorrelation function in the modified Iori model.

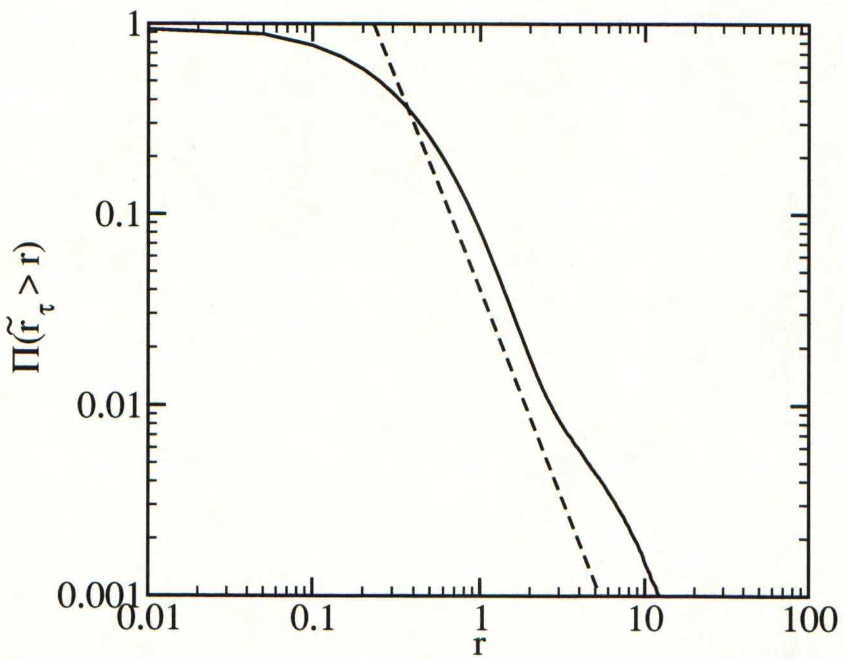


Figure 5.8: The cumulative distribution function of one step returns in the modified Iori model. The dashed line indicates the power law $r^{-2.2}$.

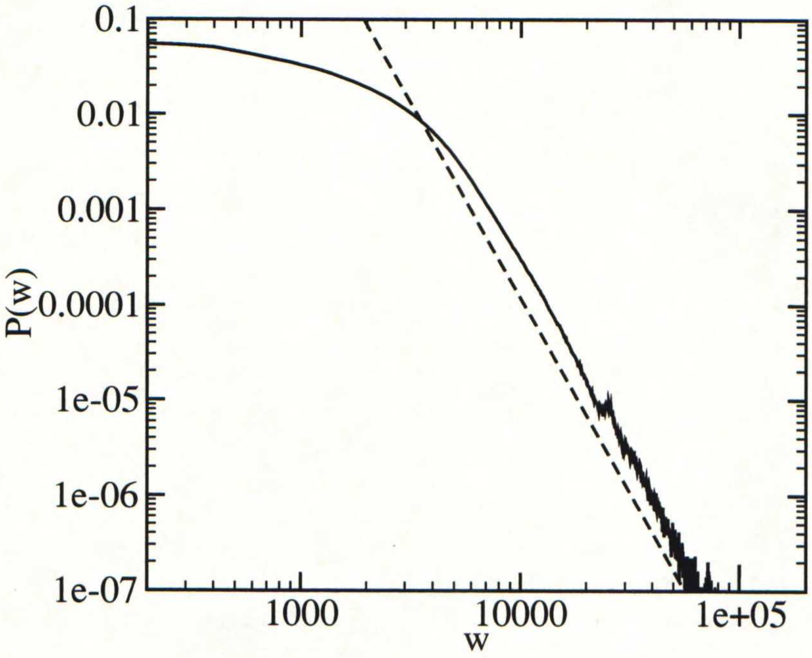


Figure 5.9: The distribution of wealth in the modified Iori model. The dashed line indicates the power law $w^{-4.1}$.

of the model. First, we tried to remove the correlation of price level and volatility due to the coupling of the trading thresholds to the price level. This we can do by allowing the trading thresholds to remain constant and instead couple the noise level A in Eq. (4.7) to the volatility or some average of the past volatilities. This would describe the increased uncertainty every agent is facing at periods of high volatility. We also tried to reverse the threshold update rule Eq. 4.13. This makes the periods of high price and high volatility correlate, which is opposite to the original behavior of the model. Otherwise the properties of the model were not changed. Secondly, an attempt was made to remove the artificial market maker and instead allow the agents to bargain with each other to establish a true equilibrium market price. However, none of these modifications could produce a volatility autocorrelation function with a power law decay, nor did they change the qualitative behavior of the model. Therefore we decided not to pursue them any further.

5.3 The Kaizoji model

5.3.1 The original model

In the Kaizoji model the only relevant parameters are the couplings J and α, and the temperature β. We chose these values according to the original article in order to reproduce those results. The values are: $J = 1$, $\alpha = 20$, and $\beta = 2.0$. The system size was again set at 100×100 agents. The other parameters appearing in the model a, b, and n_F are not relevant as they only set the scale of the price variations. We set the fundamental price $p^*(t) = 1$. In Fig. 5.10 we show the behavior of the price in this model.

First we look at the volatility autocorrelation function, which we show in Fig. 5.11. No power law is seen in this model either, but the data is in

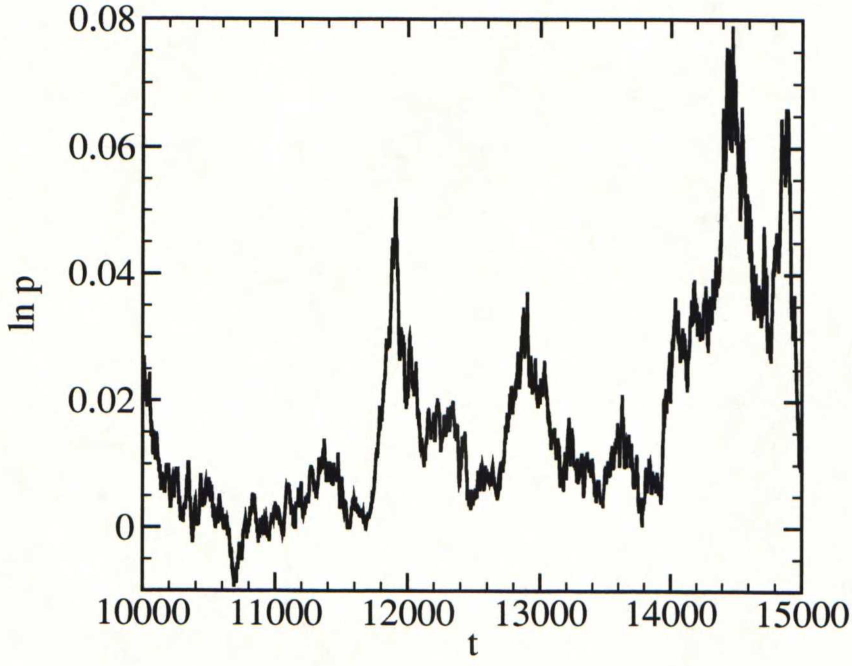


Figure 5.10: Typical behavior of the price in the Kaizoji model.

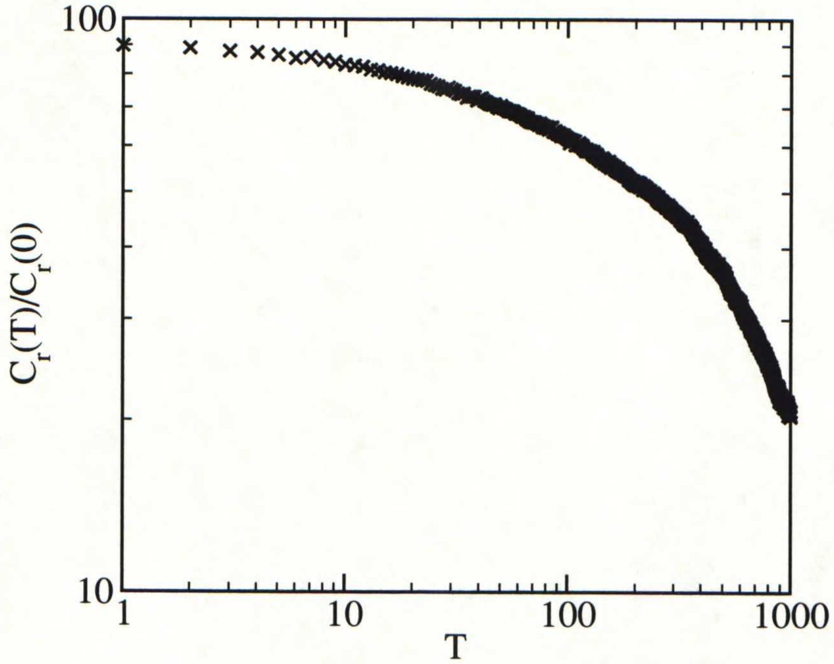


Figure 5.11: The volatility autocorrelation function in the Kaizoji model.

perfect agreement with the data presented by Kaizoji et al. in the original article.

Figure 5.12 depicts the cumulative distribution of the one step returns. A short power law regime can be seen in the middle section of the data. Also this data is in perfect agreement with the corresponding data presented by Kaizoji et al. However, as can be seen in Fig. 5.13 the tail of the distribution seems to be almost Gaussian. Thus, the price process given by this model is not in complete agreement with empirical evidence. Also in Fig. 5.13 we present the distribution of the 20000 step returns in the present model. Again, it can be seen that the distribution converges to the Gaussian distribution.

Like in the Iori model the distribution of wealth among the investors in the end of the run is perfectly Gaussian as seen in Fig. 5.14.

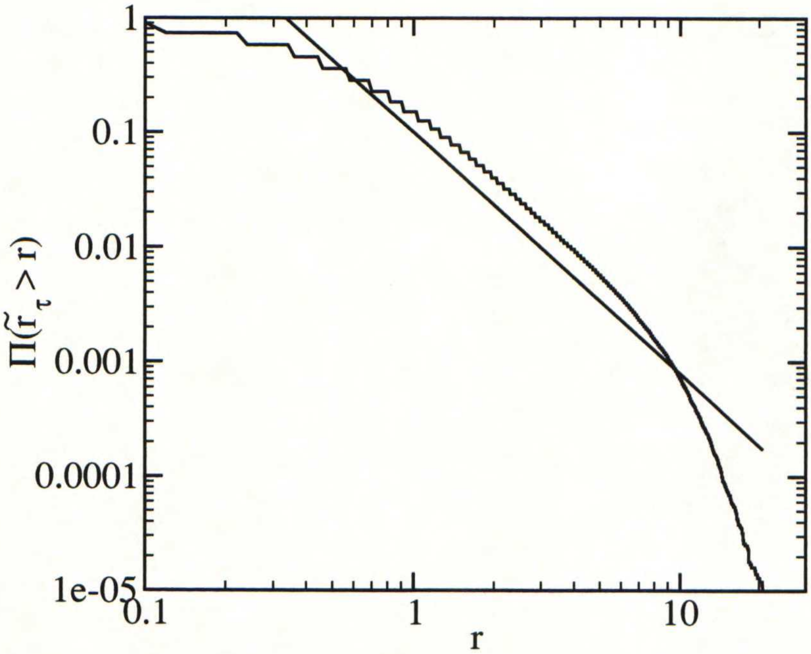


Figure 5.12: The cumulative distribution of one step returns in the Kaizoji model. The straight line shows the power law $x^{-2.1}$.

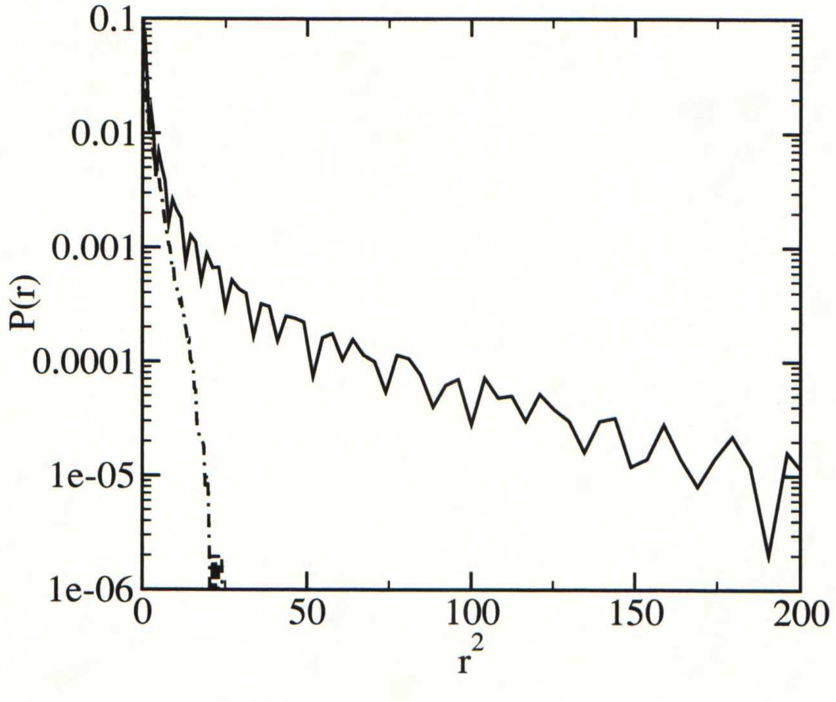


Figure 5.13: The distribution of 20000 step returns in the Kaizoji model. For comparison the solid line shows the one step returns.

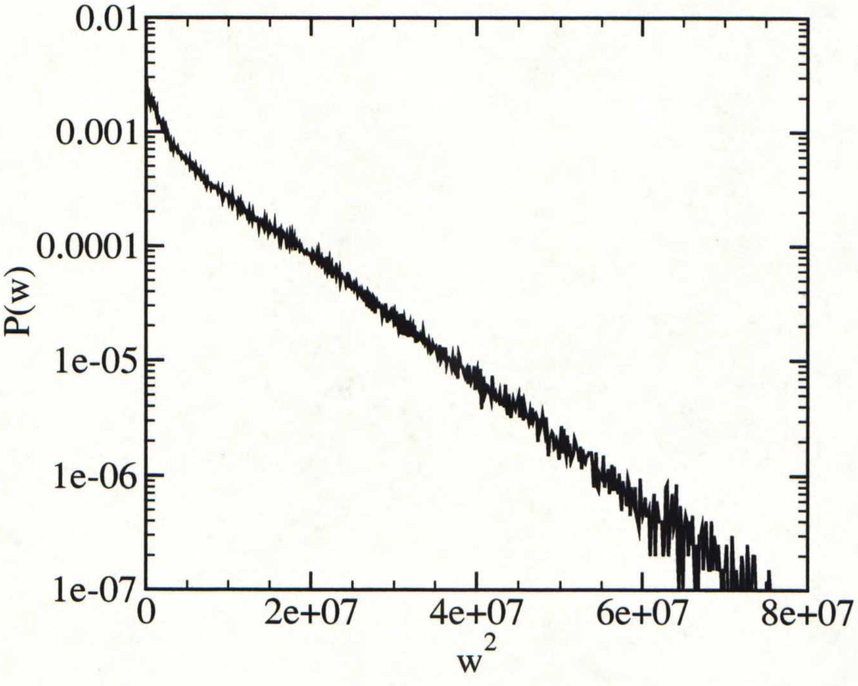


Figure 5.14: The distribution of wealth in the Kaizoji model.

5.3.2 Modified Kaizoji model

In order to sample the wealth distribution of the traders and to compare the results with the Iori model the first thing we need to do is to add the book keeping of each agent's stocks and money. This would also make it possible to implement the constraint that no agent is allowed to sell stocks he does not have or is allowed to buy stocks when he has got no money. Since this property was not included in the original model we did not implement it here either.

In the beginning of the simulation each agent has 20 units of stocks and 20 units of cash. The proportion of his wealth the agent buys or sells at one time was set to 0.01 in this model. The discussion given in connection with the Iori model above applies also here. Other parameters are kept at the same values as above for the original model.

The problem with the Kaizoji model, as already pointed out above, is that the power law in the distribution of returns spans a relatively short range. Introducing the proportional trading rule makes the range of the power law grow even shorter. We show the cumulative distribution of returns in our modified case in Fig. 5.15. Again, the observed behavior contradicts empirical findings. It needs to be kept in mind, however, that this property is not produced by our modification of the model. This problem can be seen in the results of the original model as well in Fig. 5.12. The volatility autocorrelation function remains practically the same as in the original model.

The development of the wealth distribution is extremely slow, because the scale of the price variations has to be set very low. Otherwise the system is not stable, because if the price drops to a very low value some buy or sell orders, being proportional to the agents total wealth, become very large. In Fig. 5.16 we show the wealth distribution after $6 \cdot 10^6$ time steps. The maximum of the distribution has been shifted to zero to better illustrate the

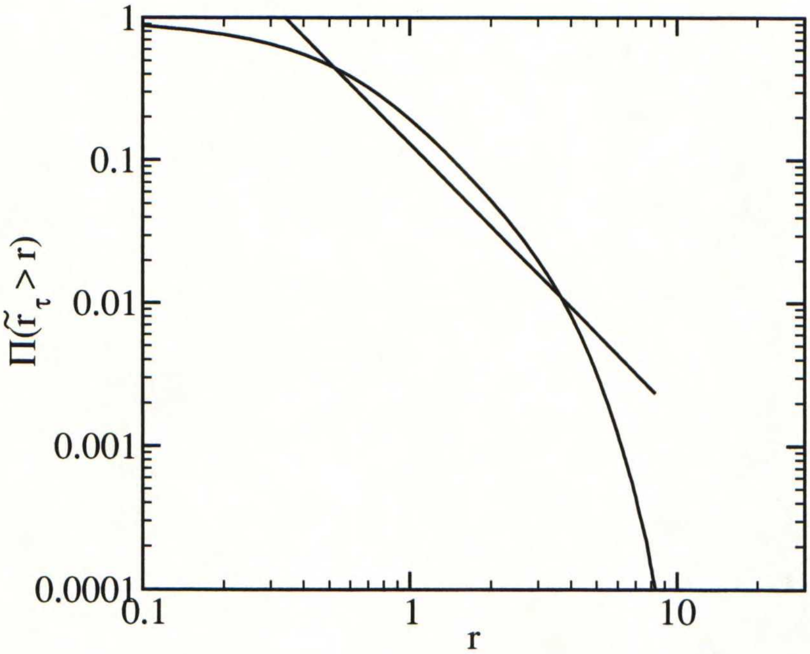


Figure 5.15: The cumulative distribution of one step returns in the Kaizoji model with proportional trading. The straight line shows the power law $x^{-1.9}$.

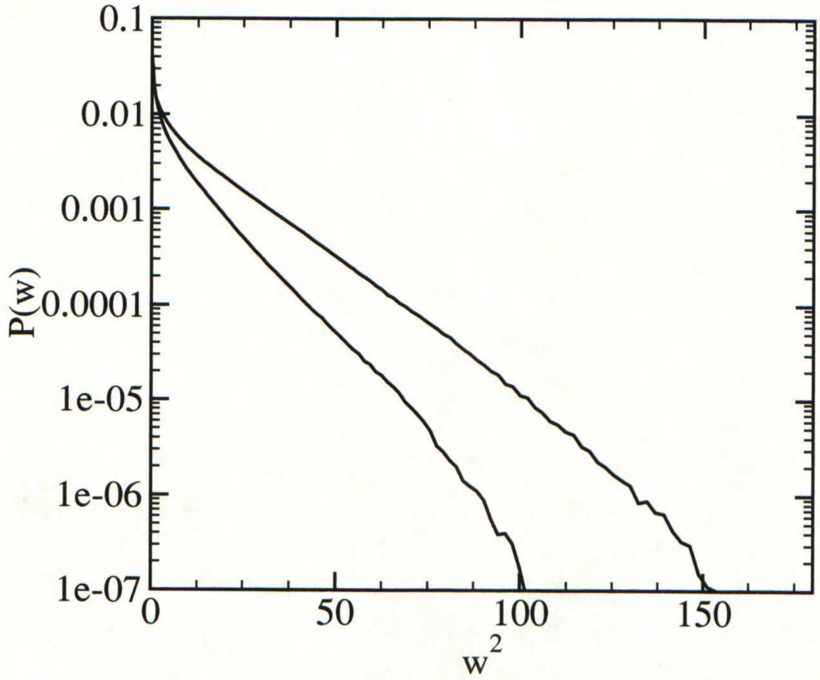


Figure 5.16: The distribution of wealth in the Kaizoji model with the proportional trading. The data have been shifted.

Gaussian nature of the distribution. There is also some asymmetry in the distribution: the left tail of the distribution is somewhat longer.

The fact that the distribution does not display power law behavior, but is rather a Gaussian can be attributed to the price process. As was seen above in Fig. 5.15 the power law nature of the price process almost disappeared in this modified model. Therefore, it is not possible to say whether the proportional trading would have been a sufficient ingredient in producing the power law wealth distribution had the price process remained unaltered.

Chapter 6

Summary and discussion

The agent based econophysics models demonstrate that the observed stylized facts may very well be produced by the trading process itself, where a number of investors interact. Naturally, this result does not rule out the possibility that the true reason is actually something more simple like the Mixed diffusion-jump model or the compound normal distribution models suggest. However, it is difficult to imagine how else some of the stylized facts like volatility clustering, gain loss asymmetry or intermittency could be explained.

We have studied two very recent models in detail. The first is the model introduced by Iori. The second is the more simple model of Kaizoji et al. The Iori model is very promising, because her results seem to reproduce many of the stylized facts observed in real stock markets. However, in our extensive analysis and simulations it turns out that the model possesses many unrealistic properties. Furthermore, one of the major merits of the model, reproducing the power law decay of the volatility autocorrelation function, turns out to be on a very weak footing. It seems to be a mere misinterpretation of the data. In our extensive simulations we did not observe this power law. However, the initial decay of the volatility autocorrelation function is such that it can easily be mistaken for a power law.

The second model of Kaizoji is an extension of the simple Bornholdt model. This model is more simple than the Iori model, but its ability to reproduce actual observed phenomena is rather limited. In particular, the distribution of the one step returns is not handled well by this model. Also this model fails to reproduce the power law decay of the volatility autocorrelation function.

The main objective of this thesis was to study the distribution of wealth among the investors in these models. It turned out that both of these models resulted in a Gaussian distribution of wealth contrary to the empirically observed Pareto power law distribution. We proposed to rectify this problem by making the agents' buy or sell order proportional to their wealth instead of always trading just one stock at a time. This was shown to result into a power law distribution of wealth in the Iori model. In the Kaizoji model, however, it was impossible to say whether this modification would have worked, because the price process was some what unrealistic to begin with and this modification only enhanced that problem.

It is interesting to note that many stylized facts of the behavior of stock prices are accurately reproduced by many different econophysical models. However, the success of none of the models proposed so far is complete. Even the best models proposed to date can only reproduce some of the stylized facts summarized in Chapter 3 of this thesis.

Empirically it has been observed that the various power law exponents discussed in this thesis seem to be somehow *universal*, that is their values are roughly the same at different markets. This would suggest that all speculative markets obey some common rules related to simple human behavior and basic market mechanisms. However, the models suggested so far lead to *non universal* exponents, that depend continuously on the values of the parameters in the model. Thus, even though in light of the recently proposed models it is reasonable to expect that several of the stylized facts are produced by collective phenomena we are only in the beginning in understanding the details of the mechanisms that give rise to them. A lot of work in this field still remains to be done.

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